Section 3.3: Increasing and decreasing functions.

Using a Graph to Determine Where a Function is Increasing, Decreasing, or Constant

A function is ***increasing*** over an open interval provided the y-coordinates of the points in the interval get larger, or equivalently the graph gets higher as it moves from left to right over the interval.

A function is ***decreasing*** over an open interval provided the $y-coordinates$ of the points in the interval get smaller, or equivalently the graph gets lower as it moves from left to right over the interval.

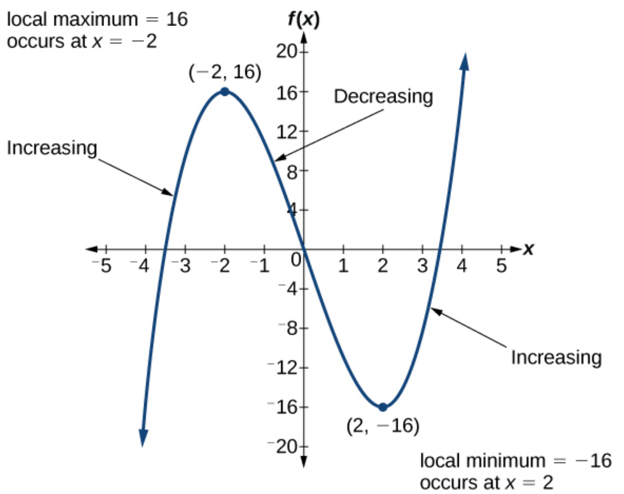
A point is a ***local maximum point*** provided it is higher than any point close to it. (Technically a point is a local maximum point if the graph changes from increasing to decreasing at that point.)

The ***local maximum value*** is the y-coordinate of the local maximum point.

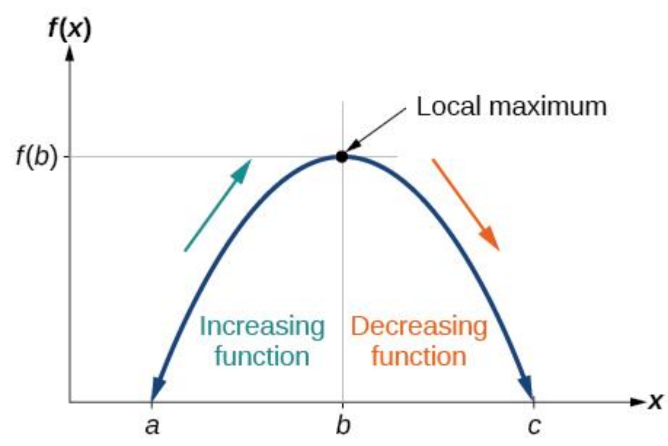
A point is a ***local minimum point*** provided it is lower than any point close to it. (Technically a point is a local minimum point if the graph changes from decreasing to increasing at that point.)

The ***local minimum value*** is the y-coordinate of the local minimum point.

Here is a visual representation of what is written above.



Here is another, example of what is written above. Notice this graph has a local maximum point, but it does not have a local minimum.



For example: Use the graph of f(x) to determine:

a) interval(s) where the graph is increasing.

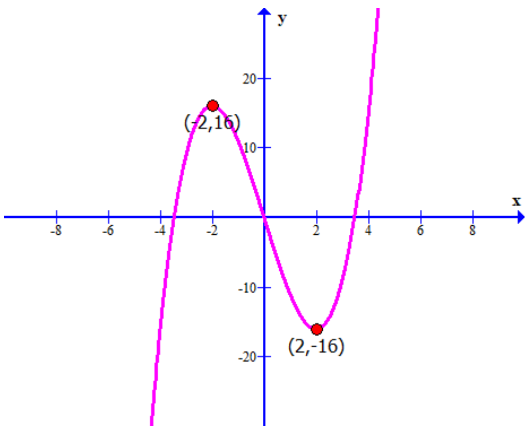
b) interval(s) where the graph is decreasing.

c) the coordinates of local maximum point, if any

d) the local maximum value

e) the coordinates of the local minimum point if any

f) the local minimum value



A point is a ***local maximum point*** provided it is higher than any point close to it. (Technically a point is a local maximum point if the graph changes from increasing to decreasing at that point.)

The ***local maximum value*** is the y-coordinate of the local maximum point.

$c)\ (-2,16)\ is\ \ a\ local\ maximum\ point,$

d) the local maximum value is $y\ =\ 16$ when x =2

A point is a ***local minimum point*** provided it is lower than any point close to it. (Technically a point is a local minimum point if the graph changes from decreasing to increasing at that point.)

The ***local minimum value*** is the y-coordinate of the local minimum point.

$e)\ (2,-16) is\ a\ local\ minimum\ point,$

f) the local minimum value is $y\ =\ -16\ when\ x = -2.$

- To determine the intervals where a graph is increasing and decreasing: break graph into intervals in terms of x, using only round parenthesis and determine if the graph is getting higher or lower in the interval.

First interval: goes from the left edge of the graph which has an $x - coordinate$ of $x = -\infty$ to the point $(-2,16)$ which has an $x - coordinate$ of $x\ = -2$

First interval $(-\infty, -2)$

Second interval goes from the point $(-2, 16)$
$x - coordinate\ x\ = -2$

to the point $(2, -16)$
$x - coordinate\ x\ = \ 2$

Second interval $(-2,3)$

Third interval goes from the point $(2, -16)$
$x - coordinate\ x\ = \ 2.$
to the right edge of the graph $x - coordinate$
$x\ = \ \infty$

Third interval $(2, \infty)$

- Determine if the graph is increasing (getting higher) or decreasing (getting lower) in each interval.

First interval $(-\infty, -2)$ increasing
Second interval $(-2,1)$ decreasing
Third interval $(2, \infty)$ increasing

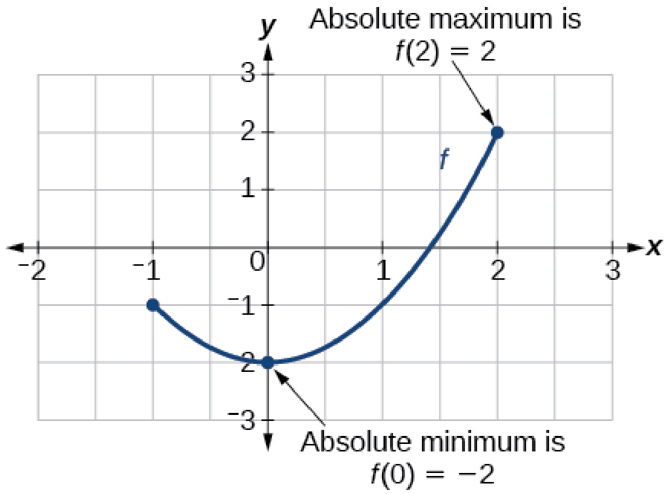
Answer: a) increasing $(-\infty, -2) \cup (2, \infty)$
b) decreasing $(-2,2)$

Use A Graph to Locate the Absolute Maximum and Absolute Minimum

- The **absolute maximum** is the highest point over the entire domain of a function or relation.
- The **absolute maximum value** is the $y - coordinate$ of the absolute maximum point.
- The **absolute minimum** is the lowest point over the entire domain of a function or relation.
- The **absolute minimum value** is the $y - coordinate$ of the absolute minimum point.

Find the

- Coordinates of the absolute maximum point.
- Value of the absolute maximum
- Coordinates of the absolute minimum point
- Value of the absolute minimum



- Coordinates of the absolute maximum point. $(2,2)$
- Value of the absolute maximum: absolute maximum value is $f(x) = 2$ which occurs when $x = 2$
- Coordinates of the absolute minimum point $(0,-2)$
- Value of the absolute minimum : absolute minimum value is $f(x) = -2$ which occurs when $x = 0$

Section 3.3: Increasing and decreasing functions.

#1 – 10: Find the

a) interval(s) where the graph is increasing.

b) interval(s) where the graph is decreasing.

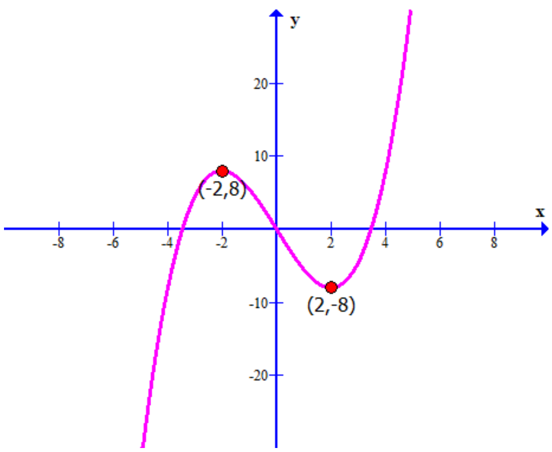
c) the coordinates of local maximum point, if any

d) the local maximum value

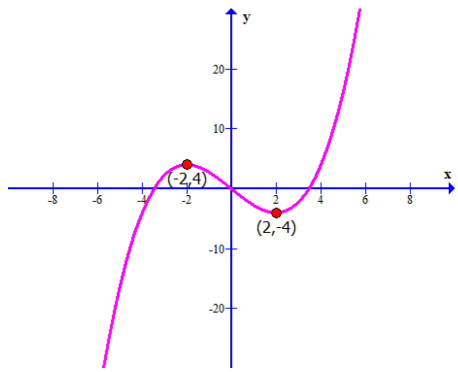
e) the coordinates of the local minimum point if any

f) the local minimum value

1)



2)



#1 – 10: Find the

a) interval(s) where the graph is increasing.

b) interval(s) where the graph is decreasing.

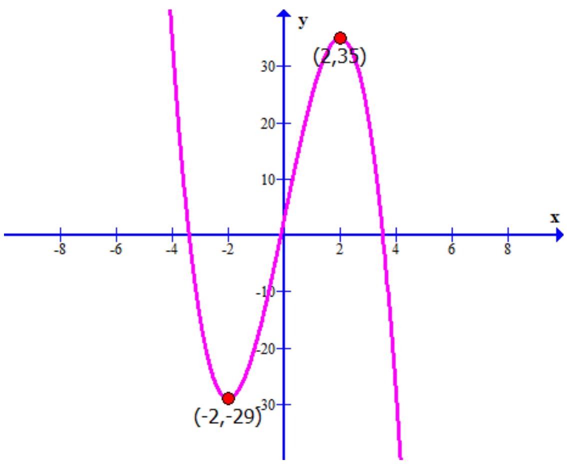
c) the coordinates of local maximum point, if any

d) the local maximum value

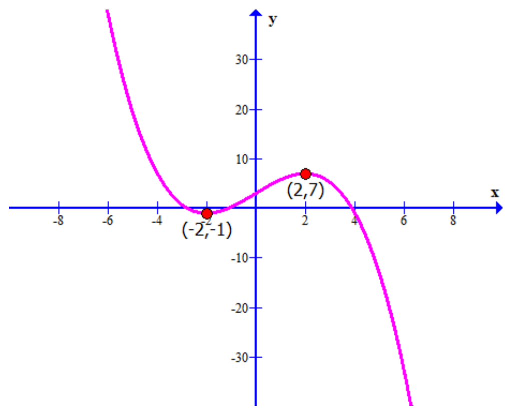
e) the coordinates of the local minimum point if any

f) the local minimum value

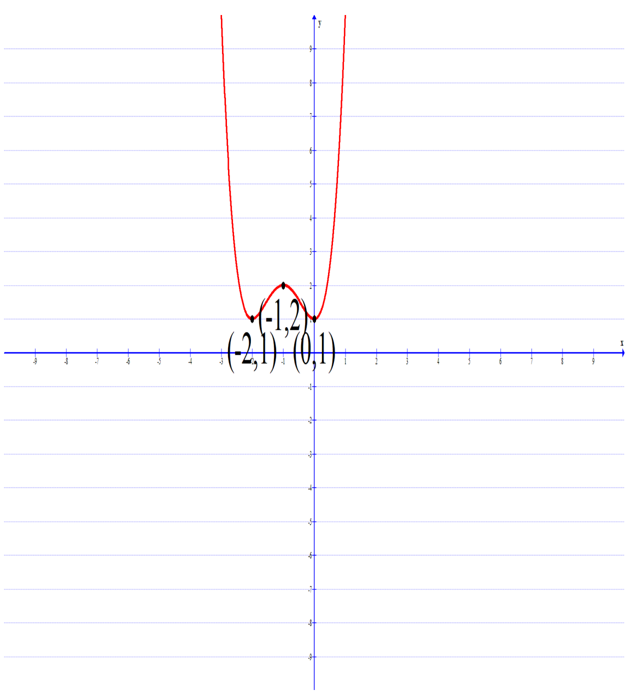
3)



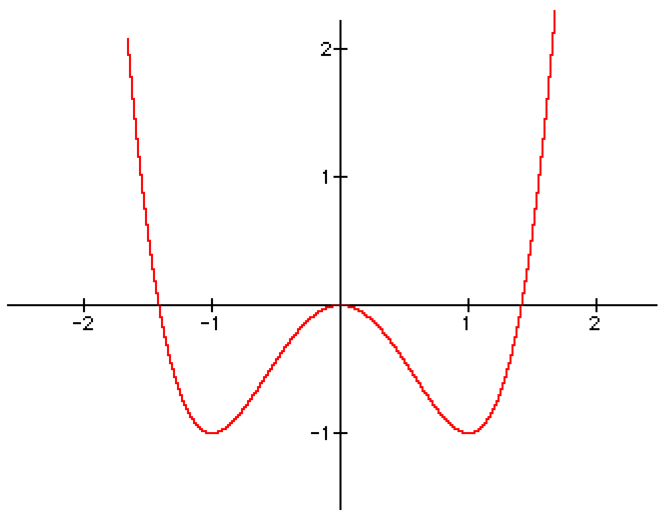
4)



5)



6)



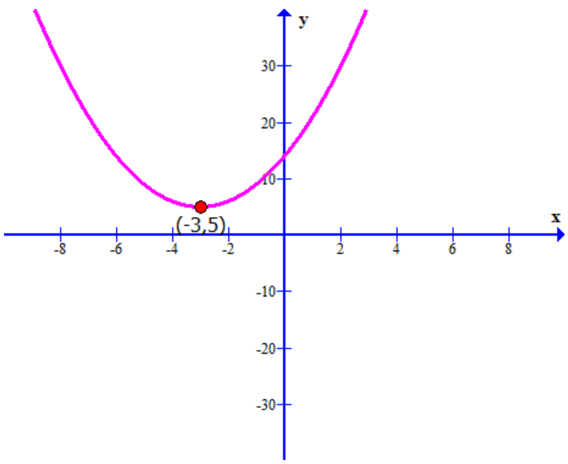
#1 – 10: Find the

a) interval(s) where the graph is increasing.
b) interval(s) where the graph is decreasing.
c) the coordinates of local maximum point, if any

d) the local maximum value

e) the coordinates of the local minimum point if any
f) the local minimum value

7)



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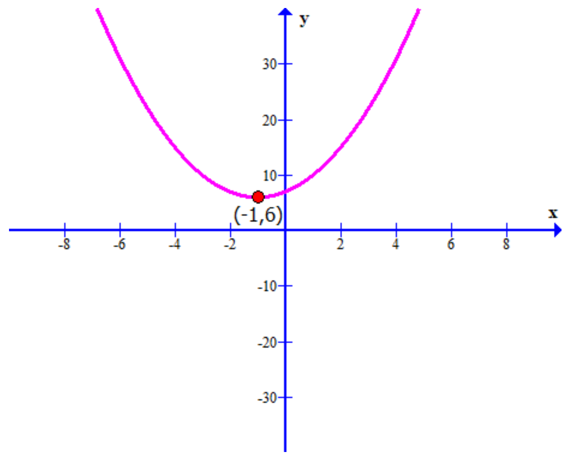

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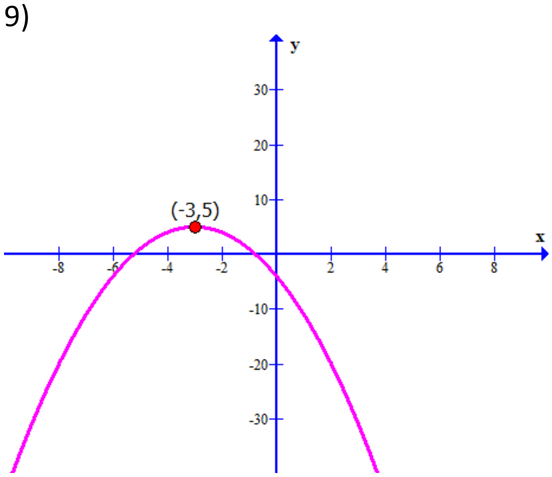
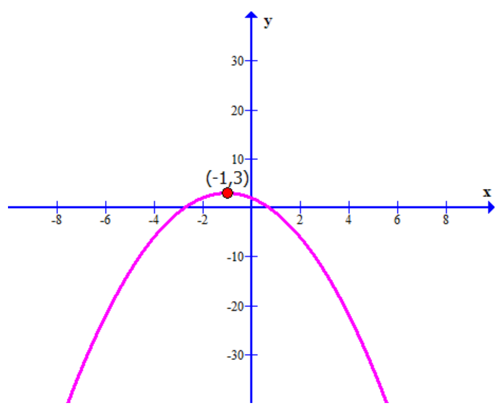
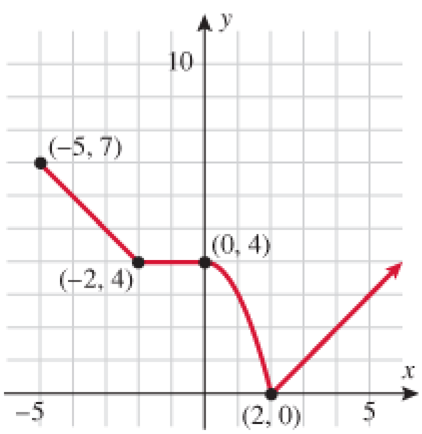
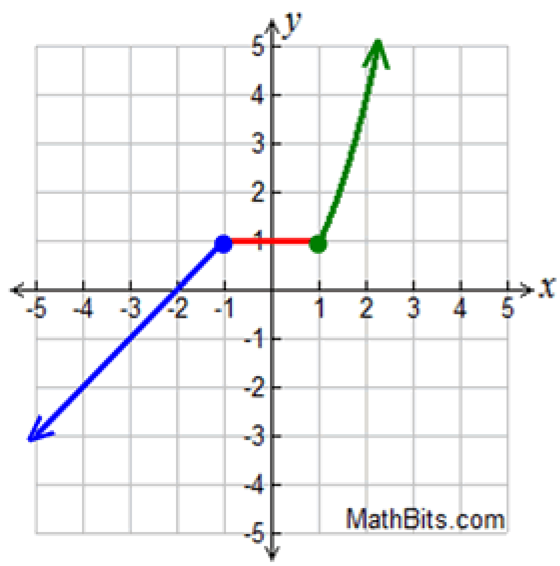


10)



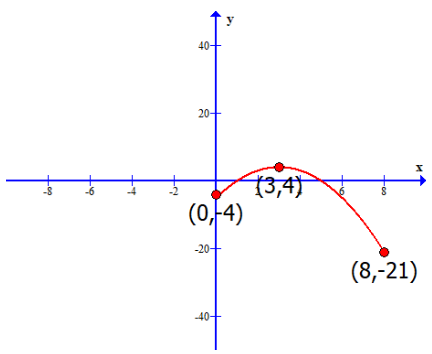
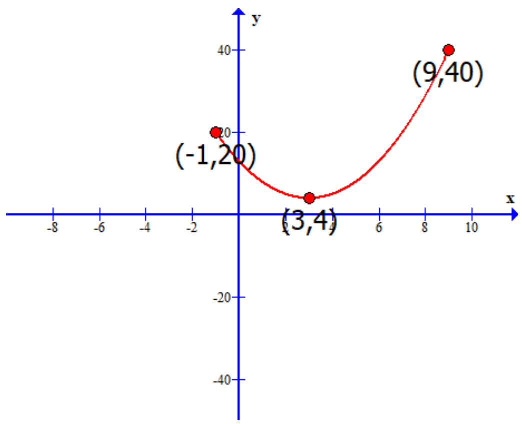
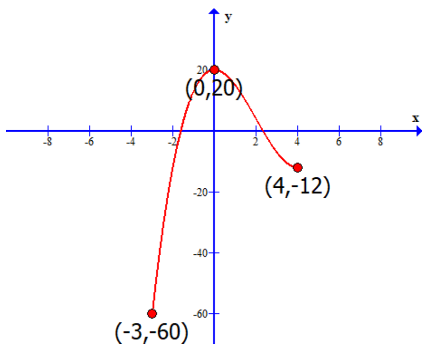
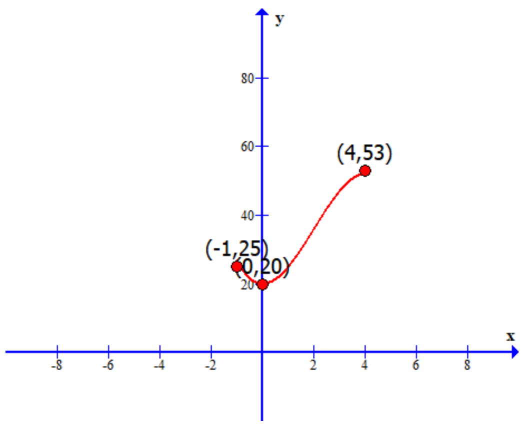
#11 – 12: Find the

a) interval(s) where the graph is increasing.
b) interval(s) where the graph is decreasing.
c) interval(s) where graph is constant
d) the coordinates of local maximum point if any
e) the local maximum value
f) the coordinates of the local minimum point if any
g) the local minimum value

11)	12)
	

#13 – 16:

a) Coordinates of the absolute maximum point.
b) Value of the absolute maximum
c) Coordinates of the absolute minimum point
d) Value of the absolute minimum

13)	14)
	
15)	16)
	

#17 – 20: Sketch a graph of the function of the indicated interval and find the following:

a) Coordinates of the absolute maximum point.
b) Value of the absolute maximum
c) Coordinates of the absolute minimum point
d) Value of the absolute minimum

17) $f(x) = (x-2)^2 + 4; [0,6]$

18) $f(x) = (x-3)^2 + 1; [1,6]$

19) $f(x) = -(x-1)^2 + 2; [-2,2]$

20) $f(x) = -(x-4)^2 + 5; [-1,5]$

21) Find the average rate of change of $f(x) = (x-2)^2 - 4$

a) from 1 to 2

b) from 3 to 5

22) Find the average rate of change of $f(x) = (x-3)^2 - 2$

a) from 1 to 3

b) from 4 to 5

23) find the average rate of change of $f(x) = x^3 - 2x + 1$

a) from -3 to -2

b) from -1 to 1

24) Find the average rate of change of $f(x) = x^3 - 3x^2 + 5$

a) from -3 to -2

b) from 4 to 6